A Game Theoretic Approach for EV Charging under Voltage Regulation Constraints

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***Abstract*—In this project, we study the behavior of Electric Vehicle (EV) charging from a game theoretic perspective. Specifically the problem of PEV charging under a smart grid environment. Moreover, the EV owners are assumed to have access to some form of renewable generation and are located at geographically different locations on a radial power distribution system. The scenario is modelled as non cooperative Stackleberg game where the utility acts as a leader and sets the price for conventional generation to maximize its profit. Each EV owner then demand energy on the basis of this price while obeying the voltage regulation norms. We first set up the mathematical formulation for such a game, and prove the existence of a Generalized Nash Equilibrium at the followers (EVs), and Stackelberg Equilibrium at the leader (grid/utility). We then illustrate the system behavior under various system settings in order to understand the interplay of the different parameters.**

I. Introduction

With the increasing shift towards alternative modes of transportation and greener sources of energy, electric vehicles (EVs) and renewable sources of energy are going to be an integral part of future power grids. However, these introduce challenges previously unseen, such as uncharacterized loads and unpredictable generation. This has encouraged the development of a bidirection communication media between the customers, the generators, and the utilities. In this project, we present an approach for solving a dynamic scenario whereby EV owners wish to purchase energy from a central utility, while having access to limited amounts of renewable sources of energy. On the utilities side, the operator wishes to meet such demands for energy, while being constrained by power flow and voltage limitations as imposed by power system protocols and transmission equipment. Moreover, the EV owners desire to minimize the expenditure in buying energy from the utility, while the utility company tries to maximize its revenue.

The idea for our project has been inspired from [1]. In this paper, the authors first formulate the problem of energy exchange between the EVs and the power grid as a static non-cooperative Stackelberg game. They then propose an algorithm that arrives at a socially optimal Stackelberg equilibrium. While the authors in [1] primarily focus on the economic aspects of EV charging, we wish to extend their work by considering an additional renewable source of energy, and considering power system constraints. The proposed extensions are as follows,

- While the utility aims to maximize profits by meeting the energy demands of the customers, the amount of power transmitted to each of the customers is subject to their location on the distribution network. Customers located far off from the feed in point might observe high voltage drops if large amount of power is allowed to flow. Meeting the voltage regulation constraints is critical from the point of view safe equipment operations.
- With more and more rooftop PV installations, the cusotmers might have access to alternate sources of energy besides the utility company. However, this source would entail its own costs which may arise out of the initial investment, equipment wear and tear, and other charges.

II. System Model

The system model for our work is extended from the one proposed in [1]. We consider a single power grid connected to multiple subscribers. The subscribers are primary — these are the main consumers such as houses, industries etc., and secondary — these are the consumers that are catered only if the power generated by the grid is sufficient to supply for all its primary subscribers and excess amount of energy is available at the grid. The PEVs that we consider in our analysis fall in the category of secondary subscribers.

We consider that the maximum amount of energy available at the power grid after catering to all its primary subscribers is C. Thus, C units of energy is available at the grid for supply to the PEVs. PEVs are grouped into a single entity, which we refer to as PEV Group (PEVG) based on their geographical locations. For example, a group of electric vehicles at a parking lot could constitute a PEVG. Then energy demands of all EVs are aggregated by the charging station at the parking lot. In our analysis, we consider that the PEVG demands energy from the grid as a single entity. There are a total of N PEVGs connected to the grid, each demanding x_n ($n \in \mathcal{N}$, $\mathcal{N}$ is the set of all PEVGs) units of energy from the grid. The battery capacity of the PEVG $n \in \mathcal{N}$ is b_n. The power grid will charge a price p per unit of energy, which is the same across all PEVGs. A small value of p will result in loss of revenue for the grid, while a high value of p can result in the PEVGs withdrawing their demand for energy. As a result, the value of p must be set by the grid so that its revenue is maximized. The demand for energy by each PEVG x_n depends on the price p set by the grid. Moreover, it is also coupled to the demand of other

PEVGs. For example, some of the PEVGs may demand large amounts of energy, resulting in a high price p set by the grid. Since the amount of energy available at the grid is limited, the aggregate demand from all the PEVGs is constrained by this limit. This translates to the following constrain,

$$\sum_{n \in \mathcal{N}} x_n \leq C \tag{1}$$

Additionally, as described in Section I, in this work, we assume that some of the PEVGs have access to a renewable source of energy (for example solar energy) that can supply energy to the vehicles in the PEVG. An example would be a solar panel installation at the parking lot where the vehicles have the option of drawing energy from the solar panel or the power grid. Hereafter, we refer to this renewable source of energy as PV. From the total requirements at a PEVG, r_n units of energy can be drawn from its PV. The cost of unit energy drawn from the PV is q, and is fixed irrespective of other factors such as time of the day, or demand for energy. q can be assumed to be the amortized cost over the lifetime of the solar panels in the PV. The maximum amount of energy available at the PV of PEVG n is assumed to be R_n (i.e. $r_n \leq R_n$). Thus, the total amount of energy demanded by the PEVG is $x_n + r_n$, where x_n and r_n are available at price p and q per unit energy respectively.

III. Formulation as a Game Theoretic Problem

The interactions between the grid and the PEVGs can be characterized as a non-cooperative game. We formulate this game as a Stackelberg game, where the grid is the leader and the PEVGs are the followers. The grid first sets the price per unit energy p, and the PEVGs then respond by demanding x_n units of energy from the grid in response to the price p and taking the price q into consideration. This stackelberg game, in its strategic form, can be defined as,

$$\Gamma = \{(\{G\} \cup \mathcal{N}), \{\mathbf{T_n}\}_{n \in \mathcal{N}}, \{U_n\}_{n \in \mathcal{N}}, L(p), p\} \tag{2}$$

where,

- the players of the game are the power grid (represented as G) and the N PEVGs,
- the strategy of each PEVG $n \in \mathcal{N}$ corresponds to the total amount of energy that uit draws from the grid and its PV, $t_n = x_n + r_n, t_n \in \mathbf{T_n}$, and is constrained by $\sum_{n \in \mathcal{N} x_n \leq C}$ and $r_n \leq R_n, n \in \mathcal{N}$,
- the utility function U_n captures the benefit of PEVG n obtained by consuming $t_n = r_n + x_n$,
- the utility function $L(p)$ is the revenue earned by the grid,
- p is the price set by the grid per unit of energy.

The utility function of the PEVG is defined similar to that in [1]. The utility function depends on the amount of energy demanded by the PEVG from the grid and its PV, the battery capacity of the PEVG, the price per unit energy set by the grid as well as the price per unit energy from the PV, and a satisfaction parameter s_n which characterizes the satisfaction that the PEVG achieves from consuming unit amount of energy. This satisfaction parameter depends on factors such as the battery capacity (a PEVG with large battery capacity will be satisfied by consuming large amount of energy), the travel plans of the PEVG, and the energy available at the PEVG at the time of plug-in. This the utility function of PEVG n can be expressed as $U_n(x_n, \mathbf{x_{-n}}, r_n, s_n, b_n, p, q)$, where $\mathbf{x_{-n}}$ is the strategy of all other PEVGs connected to the grid.

The utility function is defined from the following properties,

1) The utility function is a non decreasing function of x_n and r_n, as each PEVG will consume all the energy available until it reaches its maximum consumption level. Thus,
$$\frac{\partial U_n(.)}{\partial x_n} \geq 0, \frac{\partial U_n(.)}{\partial r_n} \geq 0.$$
2) The marginal benefit of a PEVG is a non-decreasing function of x_n and r_n as the level of satisfaction achieved by PEVG saturates as it consumes more and more energy. Thus,
$$\frac{\partial^2 U_n(.)}{\partial x_n^2} \leq 0, \frac{\partial^2 U_n(.)}{\partial r_n^2} \leq 0.$$
3) Each PEVG will try to consume as much energy as its battery capacity allows. Thus,
$$\frac{\partial U_n(.)}{\partial b_n} > 0.$$
4) A PEVG which is more satisfied by consuming a unit of energy will consume less units of energy. Therefore,
$$\frac{\partial U_n(.)}{\partial s_n} < 0.$$
5) As the price set by the grid and the price of consuming energy from the PV increases, the utility decreases.
$$\frac{\partial U_n(.)}{\partial p} < 0, \frac{\partial U_n(.)}{\partial q} < 0.$$

Considering the above requirements, the utility function is defined as,

$$U_n(x_n, \mathbf{x_{-n}}, r_n, s_n, b_n, p, q) = b_n(x_n + r_n) - \frac{1}{2}s_n(x_n + r_n)^2 - px_n - qr_n. \tag{3}$$

The utility function of the grid is simply the revenue that it generates by selling the units of power demanded by all the PEVGs. Since the price per unit energy is p, the utility of the grid is,

$$L(p, x(n)) = p \sum_{n \in \mathcal{N}} x_n. \tag{4}$$

The objective of each PEVG is to maximize its own utility once the grid sets its price p. Thus, at each PEVG, we

$$\begin{aligned} \max_{x_n} \quad & U_n(x_n, \mathbf{x_{-n}}, r_n, s_n, b_n, p, q) \\ \text{Subject to} \quad & \sum_{n \in \mathcal{N}} x_n \leq C \\ & r_n \leq R_n \\ & \text{Volatge regulation constraints} \end{aligned} \tag{5}$$

We derive the voltage regulation constraints next.

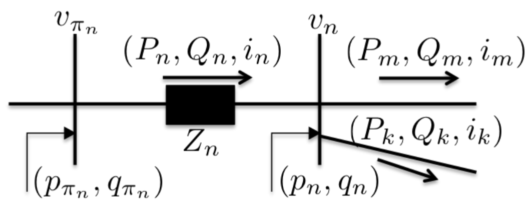


Fig. 1: Node in radial distribution grid

IV. BRANCH FLOW MODEL OF POWER SYSTEMS

In order to arrive at the voltage regulation constraints a suitable model for the power system is needed which relates the power flow through the various lines with the voltage level. For this pupose we study the branch flow model of the power system. Consider the a representative bus for the distribution grid in Fig.1. It is modelled as a radial network whereby the generators are connected to the feeder bus and the power flows out radially into downstream buses. For a siplified analysis we consider model the grid as a single phase system under the reasonable assumption that all three phases operate in a balanced conditon. Such a system can therefore by represented as a tree graph using the notation $\mathcal{G} = (\{0, \mathcal{N}\}, \mathcal{E})$ [2]. Here, $\{0, \mathcal{N}\}$ represents the set of vertices/nodes of the graph and $\mathcal{E}$ represent the set of edges. For the power system at hand the nodes correspond to the buses of the system and edges transalte to the distribution lines. Therefore for a system with $N+1$ buses a radial network will have N edges connecting them. Let $n \in \mathcal{N} = \{1, 2 \ldots, N\}$ be the index used to refer to non-feeder buses while numbering being done to enure that index for the parent bus is always less than that of the child bus. Here, let us represent the parent node for a node n by π_n. Also, let the distribution lines be number in the fashion such that edge n connects the parent child pair $\{\pi_n, n\}$.Now, consider the following definition of the various power system quanitites. For the bus n let v_n represent the square of voltage magnitude and $s_n = p_n + jq_n$ reperesent the power being injected into it, where p_n and q_n are the active and reactive power injections respectively. For the edges, let i_n be the squared magnitude of the current flowing through it, $S_n = P_n + jQ_n$ be the power flowing from node π_n to n with P_n and Q_n again representing the active and reactive components respectively, and $Z_n = R_n + jX_n$ represent the impedance. Then, it can through the given power flow relations, it can be seen that [3], [4]

$$s_n = \sum_{i \in \mathcal{C}_n} S_i - S_n + i_n Z_n \tag{6}$$

Where, $\mathcal{C}_n$ represent the set of children for node n. The first three terms in the above equation are a mere result of defined power flows whereas the last term represents the power loss because of the distribution line impedance. Next let V_n represent the voltage at node n. Then,

$$V_n = V_{\pi_n} - I_n Z_N \tag{7}$$

Multiplying the terms on both sides with their conjugates results in

$$V_n V_n^* = V_{\pi_n} V_{\pi_n}^* - V_{\pi_n}(I_n Z_n)^* - V_{\pi_n}^* I_n Z_n + (I_n Z_n)(I_n Z_n)^* \tag{8}$$

Further, noticing that $S_n = V_{\pi_n} I_n^*$ and using the definition of v_n and the above equation changes to

$$v_n = v_{\pi_n} - S_n Z_n^* - S_n^* Z_n + i_n |z_n|^2 \tag{9}$$

$$v_n = v_{\pi_n} - 2\text{Re}[S_n Z_n^*] + i_n |z_n|^2 cx \tag{10}$$

Next, introducing $\mathbf{A}$ as the reduced branch-bus incidence matrix. The entries of $\mathbf{A}$ given by a_{ij} are set to 0 if the branch is not connected to bus j. Otherwise, it is set to 1 or -1 depending upon whether the node j is a parent or a child node respectively. Further, introducing $\mathbf{v} = [v_1, v_2, \ldots, v_N]^T$, $\mathbf{p} = [p_1, p_2, \ldots, p_N]^T$, $\mathbf{q} = [q_1, q_2, \ldots, q_N]^T$, $\mathbf{P} = [P_1, P_2, \ldots, P_N]^T$, $\mathbf{Q} = [Q_1, Q_2, \ldots, Q_N]^T$, $\mathbf{R} = [R_1, R_2, \ldots, R_N]^T$ and $\mathbf{X} = [X_1, X_2, \ldots, X_N]^T$. FInally $\mathbf{s} = \mathbf{p} + j\mathbf{q}$, $\mathbf{S} = \mathbf{P} + j\mathbf{Q}$ and $\mathbf{Z} = \mathbf{R} + j\mathbf{X}$. Now using the following approximations [3], [4] $i_n Z_n \approx 0$ and $i_n |Z_n|^2 \approx 0$ in (6) and (10) respectively gives [5]

$$\mathbf{s} = \mathbf{A}^T \mathbf{S} \tag{11}$$

$$\mathbf{A}\mathbf{v} = 2\text{Re}[\mathbf{Z_d^*}\mathbf{S}] - \mathbf{a_0} v_0 \tag{12}$$

Where $\mathbf{Z_d} = \text{diag}(\mathbf{Z})$ and $\mathbf{a_0}$ completes the reduced brach-bus incidence matrix by including conections for the feeder bus. Defining $\mathbf{A}' = -A^{-1}$. Now from (11), $\mathbf{S} = -\mathbf{A}'^T\mathbf{s}$. Substituting this in (12) and premultiplying it by $-\mathbf{A}'$ gives

$$\mathbf{v} = \mathbf{R_d}\mathbf{p} + \mathbf{X_d}\mathbf{q} + v_0 \mathbf{1_N} \tag{13}$$

In arriving at (13) certain properties of $\mathbf{A}'$ have been used which have been omitted here for brevity. Here, $\mathbf{R_d} := 2\mathbf{A}'\text{diag}(\mathbf{R})\mathbf{A}'^T$ and $\mathbf{X_d} := 2\mathbf{A}'\text{diag}(\mathbf{X})\mathbf{A}'^T$. FInally, v_0 is the voltage of the feeder node.

V. VOLTAGE REGULATION CONSTRAINT

Having derived the final relation for the branch flow model in the previous section, (13) can be used to impose voltage regulation constraints that restrict the deviation of voltage magnitudes from the nominal value at feeder node v_0. Specifically a constraint of the form $-\alpha \leq v_n - v_0 \leq \alpha$ for a positive α and for all the bus nodes can be translated to the following using (13)

$$-\alpha \mathbf{1_N} \leq \mathbf{R_d}\mathbf{p} + \mathbf{X_d}\mathbf{q} \leq \alpha \mathbf{1_N} \tag{14}$$

This is the required power system constraint that all the demands in (2) will have to be satisfied. For this case the elements of $\mathbf{p}$ will be composed of active power demands from the PEVs as well as conventional loads on the system given by p_n^ℓ whose values are pre-determined. Similarly, the reactive power demand $\mathbf{q}$ is composed of sums of that from PEVs and other predetermined loads (q_n^ℓ). For simiplicty we assume that the renewable energy r_n is active in nature and is directly integrated at bus n and therefore does not participate in the network constraints. The power demand from the grid x_n is also active in nature but is accompanied with an additional reactive power demand which is given by $x_n tan(cos^{-1}(\text{P.F.}))$ where P.F. is the power factor of PEV charging equipment. With these the following relations define the elements of the vectors $\mathbf{p}$ and $\mathbf{q}$

$$p_n = x_n + p_n^{\ell} \tag{15}$$

$$q_n = x_n tan(cos^{-1}(\text{P.F.})) + q_n^{\ell}. \tag{16}$$

VI. Existence of Generalized Stackelberg Equilibrium (GSE)

In this section, we show the existence of the Generalized Stackelberg Equilibrium for the game Γ. We prove the existence of the GSE in two steps — first, we show that for a given price p set by the leader, there exists a GNE for the followers, (ii) second, we show that for the followers' GNE solution, there exists a Stackelberg Equilibrium for the leader.

A. Existence of Followers' Generalized Nash Equilibrium

The strategy of each follower are the tuples x_n and r_n, i.e. the energy demanded from the utility and renewable source, respectively. Let us define the vector $v_n = [x_n \;\; r_n]^T, v_n \in \mathbb{R}^2$. For each player $n \in \mathcal{N}$, the problem of maximizing its payoff is equivalent to minimizing the negative payoff function,

$$\begin{aligned} V_n &= -U_n = v_n^T Q_n v_n + c_n^T v_n \\ \text{where, } Q_n &= \begin{bmatrix} 0.5s_n & 0.5s_n \\ 0.5s_n & 0.5s_n \end{bmatrix} \\ \text{and } c_n &= \begin{bmatrix} p - b_n \\ q - b_n \end{bmatrix} \end{aligned} \tag{17}$$

We define $v = [v_1; v_2; \cdots ; v_N]^T$ as the stacked vector of strategies of all the N players. Thus, $v \in \mathbb{R}^{2N}$. For each player, the strategy vector has a local constraint (for r_n) which is not coupled with strategies of other players in the system, and several coupled linear constraints (for x_n). We encompass the local and global constraints of the game Γ into a single set of feasible solutions, denoted as $\mathcal{V}$. Thus,

$$v \in \mathcal{V} := \{v \in \mathbb{R}^{2N} | Av \leq b\} \subset \mathbb{R}^{2N}. \tag{18}$$

where $A \in \mathbb{R}^{m \times 2N}$ and $b \in \mathbb{R}^m$.

Observation 1: The satisfaction parameter $s_n \geq 0$. Hence, the matrix Q_n is positive semidefinite. Thus, the payoff function for each player is convex.

Definition 1: Generalized Nash Equilibrium A vector v^* is called a GNE of the game if $v^* \in \mathcal{V}$ and $V_n(v_n^*, \mathbf{v}_{-\mathbf{n}}^*) \leq V_n(v_n, \mathbf{v}_{-\mathbf{n}}) \quad \forall v_n \in \mathcal{V}$. This class of NE is called the GNE because the strategy of each player depends on the strategies of other players in the system.

Definition 2: Variational Equilibria The GNE solution for the game Γ is closely related to the solution of the variational inequality (VI) problem [6]. We first define the VI problem. Consider an operator $T(.) : \mathbb{R}^d \rightarrow \mathbb{R}^d$ and a set $\mathcal{Y} \subseteq \mathbb{R}^d$. Then, the solution to the VI problem VI($\mathcal{Y}, T$) is a vector $y^* \in \mathcal{Y}$ such that $\langle T(y^*), y - y^* \rangle \geq 0 \quad \forall y \in \mathcal{Y}$. The VI problem is guaranteed to have a solution under certain conditions which will be described next. Moreover, the solution set of the VI need not be singleton, i.e. the VI can exhibit multiple solutions.

It has been shown that the set of solutions of VI($\mathcal{V}, F$) is a subset of the solutions of the GNE of the game Γ [1], [6], where the operator F is defined as,

$$\begin{aligned} F(v) &:= [\Delta_{v_n} V_n(v_n, \mathbf{v}_{-\mathbf{n}})]_{n=1}^N \\ &= F_1 v + f_2 \\ \text{where, } F_1 &:= \text{diag}\left(Q_n + Q_n^T\right) \\ \text{and } f_2 &:= [c_1; c_2; \cdots ; c_N]^T. \end{aligned} \tag{19}$$

Thus, in order to prove the existence of the GNE, it suffices to prove that a solution to the problem VI($\mathcal{Y}, T$) exists.

Definition 3: Monotone operator The operator $T(.)$ is said to be a Monotone operator iff $\langle T(x) - T(y), x - y \rangle \geq 0 \quad \forall x, y \in \mathbb{R}^d$.

Proof of existence of VI (and GNE): Theory of VI states that if the operator $F(.)$ is monotone, then the set of solutions of VI($\mathcal{V}, F$) is non-empty [6], [7]. Moreover, the operator $F(.)$ defined in Eq. (19) is monotone iff the matrix $\frac{F_1 + F_1^T}{2}$ is positive semidefinite. This condition is easy to check, and the operator $F(.)$ is indeed monotone. Thus, the set of solutions of VI($\mathcal{Y}, T$) is non-empty. Therefore, GNE for the game Γ always exists. ∎

Observation 2: It must be noted that since the matrix $\frac{F_1 + F_1^T}{2}$ is not positive definite, the VI can exhibit multiple solutions.

B. Existence of Leader's Stackelberg Equilibrium (SE)

The strategy of the leader is the price p it sets per unit energy. The Stackelberg Equilibrium for the leader exists if the strategy set of the leader is closed and compact [8]. Thus, it suffices to show that the strategy set of the leader, i.e. p is closed and compact.

Proof of existence of SE: Clearly, the price p is lower bounded by 0. We now show that, for the leader, the price p cannot be unbounded.

Consider the payoff of the follower $n \in \mathcal{N}$. For simplicity, we let the demand of follower n from the renewable source be 0. In this case, the payoff of player n is given by,

$$U_n = b_n x_n - \frac{1}{2} s_n x_n^2 - p x_n$$

Figure 2 shows player n's strategy versus its payoff. Clearly, when $x_n = 0$, $U_n = 0$. The utility is maximum when $x_n = \frac{b_n - p}{s_n}$, and the corresponding utility of the player is $U_n = \frac{(b_n - p)^2}{2s_n}$. Thus, as the price p increases to b_n, the downward facing parabola shown in Figure 2 degenerates to the point $(0, 0)$. For any further increase in p, the demand from player n is 0, and the payoff of the grid with respect to player n is 0. Let $b_{max} = \max\{b_1, b_2, \cdots , b_N\}$. If $p > b_{max}$, the payoff of the grid $L(p) = 0$. Let $p_{max} = b_{max}$. The strategy set of the utility is upper bounded by p_{max}. Thus, $p \in [0, p_{max}]$. The strategy set of the utility is closed and compact, and a Stackelberg Equilibrium for the leader exists. ∎

The existence of followers' GNE and leader's SE concludes that a Generalized Stackelberg Equilibrium exists for the

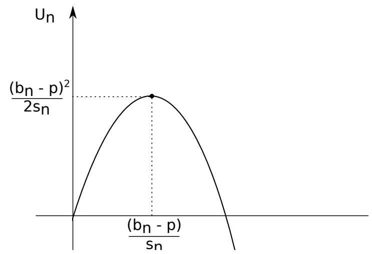


Fig. 2: Utility of player n, U_n as a function of its strategy x_n

system. We describe a method to solve the problem of finding the GNE for the followers next.

C. A Centralized Solution to the GNE

In this subsection, we show that the problem of achieving the maximum utility at each player n individually is equivalent to maximizing a single convex quadratic utility function.

Proposition 1: The game Γ is a potential game with potential function given by,

$$P(v) = \frac{1}{2}v^T F_1 v + f_2^T v \tag{20}$$

where F_1, f_2 are as defined in Sec. VI-A.

Proof: It follows from the definition of $P(v)$ that $\Delta_{v_n} P(v) = \Delta_{v_n} V_n(v_n, \mathbf{v_{-n}})$. Moreover, it is easy to verify that for fixed strategies of remaining followers $\mathbf{v_{-n}}$, $V_n(v_n^1, \mathbf{v_{-n}}) - V_n(v_n^2, \mathbf{v_{-n}}) = P(v_n^1, \mathbf{v_{-n}}) - P(v_n^2, \mathbf{v_{-n}})$. Thus, the game Γ is an exact potential game with potential function $P(v)$ as defined by Eq. (20). ■

The above proposition enables solving the followers' game using a single convex optimization problem at a central entity.

VII. Experimental Results

In this section, we simulate a small-scale power distribution system in order to provide proof-of-concept of the existence of the GSE in a given system with 1 leader and 4 followers.

A. Experimental Topology

We use a simple topology with one generator bus and four buses for distribution. We use the IEEE 5 bus radial system. We omit the details of the topology for the sake of saving space. At each distribution point, a primary load demands energy from the grid. At the same time PEVGs demand x_n amount of energy from the grid. The total battery capacity of each PEVG is taken to be the aggregate battery capacity of each PEVG. We assume that the battery capacity of each EV is 65 kWh (the battery energy rating of a Tesla EVs). For all the results in the subsequent sections, we assume that there are a total of 100 EVs in each PEVG. The satisfaction parameters of each PEVG is fixed to 1.

B. Results

1) Impact of Power System Constraints: We start with the observation that under realistic settings, the load of supporting PEVGs is comparable (if nor larger than) the primary load in the grid. This leads to the consequence that the energy demanded by the PEVGs may not be feasible to provide under the power system constraints. To illustrate this fact, we observe the energy supplied to the PEVGs with and without power system constraints.

We use the system parameters as shown in Case I of Table I and Table II with $q = \$1$. At the GNE, the followers each demand $x_1 = x_2 = x_3 = x_4 = 2.755$ and $\sum x_n = 11.02$. On the other hand, with the power system constraints in effect, $\sum x_n = 2.53$. This shows that the power system constraints can have significant impact on the energy drawn from the grid. Simply because a PEVG requires certain amount of energy, supplying this energy from the grid may not be feasible under the power system constraints.

2) Impact of System Parameters: The EVGs continue to draw power from the grid and/or the renewable source until the total demanded energy exceeds their respective battery capacity. We consider a few representative cases of the total availability of energy at the utility (i.e. C) and the renewable source (i.e. $\sum R_n$) in order to illustrate the behavior of the system under different settings. As we see in the following discussions, the energy demanded from the two sources varies depending on the price per unit energy set by the utility as well as the renewable source. The parameter values set for each case are listed in Table I.

Case	b_n	s_n	C	R_n
I	[6.5,6.5,6.5,6.5]	[1,1,1,1]	15	[1,1,1,1]
II	[6.5,6.5,6.5,6.5]	[1,1,1,1]	35	[8,8,8,8]
III	[6.5,6.5,6.5,6.5]	[1,1,1,1]	35	[1,1,1,1]
IV	[6.5,6.5,6.5,6.5]	[1,1,1,1]	1	[8,8,8,8]
V	[6.5,6.5,6.5,6.5]	[1,1,1,1]	1	[1,1,1,1]

TABLE I: EVG parameter values

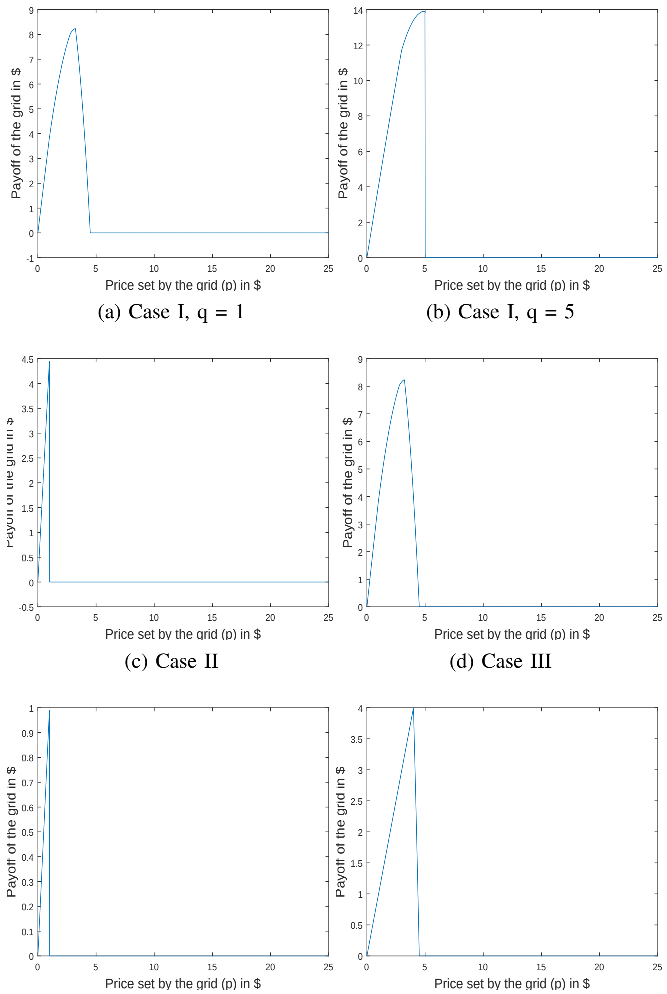
Case 1: $C + \sum R_n < \sum b_n$, i.e. the total energy available at the two sources is less than the cumulative battery capacity of the PEVGs.

We first set $q = \$1$ and vary p from \$0 to \$25. In this case, the PEVGs draw the maximum power from the renewable source irrespective of the price p set by the utility, whereas the demand from the grid depends on p. The payoff of the grid is maximized when $p = \$3.22$ as seen in Figure 3a.

Next, we repeat the above setting with $q = \$5$. In this case, since the price set by the renewable source is high to begin with, drawing energy from the renewable source has large negative impact on the payoff of the EVGs. Thus, despite needing the energy, the EVGs do not draw the maximum power from the renewable source. In this case, the payoff of the grid is maximized (Figure 3b) when $p = \$5 (p = q)$. The values of x_n and r_n at the equilibrium for all cases are shown in Table II ($q = \$1$ in each case).

Case 2: $C > \sum b_n, \sum R_n > \sum b_n$, i.e. energy available at each source is sufficient to cater to the PEVG demands.

We set $q = \$1$. The payoff of the grid is maximum when $p = q = \$1$ as shown in Figure 3c. The PEVGs draw no or little power from the renewable source (depending on its position in the grid) when $p < q$. As $p > q$, PEVGs continue



(a) Case I, q = 1
(b) Case I, q = 5
(c) Case II
(d) Case III
(e) Case IV
(f) Case V

Fig. 3: Payoff of grid as a function of p

to draw power from renewable sources until any more power has a negative impact on their utilities.

Case 3: $C > \sum b_n, \sum R_n < \sum b_n$.

Case 4: $C < \sum b_n, \sum R_n > \sum b_n$.

Case 5: $C < \sum b_n, \sum R_n < \sum b_n$.

The payoff of the grid in each of the cases are shown in Figure 3. Discussions on each of the cases are omitted due to the lack of space. However, the general trends are listed in the observations as follows,

Case	x_1	x_2	x_3	x_4	r_1	r_2	r_3	r_4
I	0.63	0.64	0.63	0.63	1	1	1	1
II	2.72	1,76	0	0.02	0.02	0.98	2.75	2.73
III	0.63	0.64	0.63	0.63	1	1	1	1
IV	0.18	0.46	0.17	0.17	2.56	2.28	2.57	2.57
V	0.25	0.25	0.25	0.25	1	1	1	1

TABLE II: Energy drawn by the PEVGs at equilibrium

Observations:

- In each case, there is a unique Generalized Stackelberg Equilibrium, which maximizes the payoff of the grid as well as the PEVGs.
- The payoff of the grid depends not only on the price p it sets, but also on the price q set and the energy available at the renewable source. If the energy available at the renewable source is greater than that required by the PEVGs, the price q acts as a strict higher bound on p. However, when the demand is much higher than the supply, the utility can set its price higher than q.
- When q is high, the EVGs may not draw the maximum power from the renewable source despite the need.
- If $\sum b_n < \sum R_n$, the Stackelberg equilibrium for the leader strictly lies within the range $0 \leq p \leq q$. However, $\sum b_n > \sum R_n$, the leader can set a price higher than q.
- Results of Case I and III are identical in each case as the value of C, despite being large, power system constraints do not allow drawing large power from the grid.

VIII. Conclusions

In this project, we extended the work carried out in [1] by including power system constraints and integrating a renewable energy source for the problem of EV charging from a game theoretic perspective. The game was formulated as a Stackelberg game with the grid as the leader and the EVs as the followers. We proved that under the proposed system model, the game the guaranteed to exhbit a Generalized Stackelberg Equilibrium solution, i.e. a Stackelberg Equilibrium exists for the leader and a Generalized Nash Equilibrium exists for the followers. We include experimental results to understand the impact of power system constraints on the system performance, and the interplay between different system parameters.

As a part of this project, we tried to implement a distributed algorithm for achieving the GSE solution. The algorithm was based on simple best response dynamics, where each follower tries to maximize its utility given other followers' strategies. However, the algorithm did not converge. As a part of the future work, we plan to propose and implement a mathematically sound algorithm for arriving at the GSE solution in a distributed fashion. We wish to submit the resulting work in a conference publication.

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