A Computationally Efficient Node-Selection Scheme for Cooperative Beamforming in Cognitive Radio Enabled 5G Systems

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***Abstract*—*Cooperative transmit beamforming* (CTB) is a practical approach for addressing the challenging problem of spectrum scarcity in broadband 5G wireless communication systems. It is a technique that allows a group of *secondary users* (SUs), each equipped with a single omni-directional antenna, to collaborate and steer the signal towards the intended receiver. CTB allows SUs to co-exist with *primary users* in the same spectrum, which helps to significantly improve the efficiency of spectrum utilization. One of the key factors that affect the performance of CTB is the selection of participatory nodes. In this paper, we first formulate the CTB as an optimization problem, and then investigate the impact of different node-selection schemes on the performance of CTB. Our findings illustrate that exhaustive search based optimal node-selection scheme is computationally infeasible for real-time systems, while simple random-selection and highest-channel-state based selection often result in poor performance. Motivated by these findings, we propose a computationally efficient node-selection scheme for CTB that achieves a near-optimal performance. The proposed scheme is based on iterative node-replacement and is computationally scalable to large system size. Results from extensive simulations show that the proposed scheme asymptotically approaches the exhaustive search based optimal system performance. In our example, the performance of the proposed scheme is approximately 98.5% of the optimal system performance while limiting the required computations to only 1.67%.**

I. Introduction

In recent years, the wireless industry and academia have been focusing on the research of Fifth Generation (5G) wireless systems, which are expected to meet the explosively growing demands for broadband mobile applications. One of the fundamental research challenges in developing 5G systems is the inadequate availability of spectrum in the radio band. *Cognitive Radio* (CR) [1]—a device or a network that dynamically adapts to the radio environment—is an emerging technology to tackle, or at least partly address, the aforementioned challenge in 5G cellular networks. As radio spectrum becomes more crowded, the hope of CR in improving the spectral efficiency has attracted significant research interests. In a CR network (CRN), there are two approaches by which unlicensed users (a.k.a. *secondary users* (SUs)) can coexist with licensed users (a.k.a. *primary users* (PUs)) in the same spectrum. SUs can opportunistically access the radio spectrum when PUs are not using it—this is referred to as the *overlay* system. SUs can also concurrently use the same spectrum, if they can ensure that the interference towards the PU is below a predefined threshold—this is referred to as the *underlay* system. In this paper, we focus on underlay CR system.

Transmit beamforming [2]–[5] is an effective mechanism that enables the operation of SUs in an underlay CRN by exploiting *spatial white-spaces*. It is a technique in which a SU transmitter (SU-Tx) equipped with multiple antennas can steer the transmit signal towards the intended SU receiver (SU-Rx) while constraining the signal power in the direction of the PUs. However, in certain scenarios, such as nodes in a wireless sensor network and other Internet-of-Things (IoT) applications, a transmitter may only be equipped with a single omni-directional antenna, and hence, it may not be able to implement beamforming on its own. An alternative way to realize beamforming in such cases is to utilize a "virtual" antenna array, which can be created by a set of cooperative nodes [6]–[9]. This technique—often termed as *cooperative beamforming* or *distributive beamforming*—allows multiple single-antenna-transmitters to collaborate and steer a common message signal towards the desired receiver while limiting interference in the direction of co-channel PUs.

Previous work on cooperative beamforming mainly focused on addressing issues related to careful collaboration and timely information-sharing among the participatory nodes. In [8], a collaborative beamforming technique was proposed in which randomly distributed nodes in a CRN form an antenna array and beamform signal to a faraway destination without each node exceeding its power constraint. The design of a distributed zero-forcing beamformer in a relay-assisted CRN was proposed in [10]. The design of cooperative beamforming that is robust against uncertainties in steering vector in CRNs was investigated in [11]. In [12], it was shown that randomly distributed nodes can achieve a nice average beampattern with a narrow main lobe and low side lobes.

Often in CRNs, several practical constraints, such as constraints related to synchronization and timely information-sharing, limit the number of nodes that can participate in cooperative beamforming. When the number of available nodes is larger than the maximum number of nodes that can participate in beamforming, the selection of nodes is critical to the performance of cooperative beamforming. However, to the best of our knowledge, the effect of different node-selection schemes on the performance of cooperative beamforming in a large CRN has not been studied in detail yet. We make the following contributions in this paper.

- We investigate how different node-selection schemes affect the performance of cooperative beamforming in a CRN. Our findings illustrate that the optimal node-selection scheme—i.e., the scheme that achieves the maximum signal-to-noise ratio (SNR) at the intended receiver—is computationally expensive, and such a scheme is not feasible to implement in large wireless networks (networks with large number of nodes).
- Motivated by the aforementioned findings, we propose a computationally efficient scheme for selecting nodes in cooperative beamforming. The proposed scheme achieves a near-optimal performance, and it can also be applied to large CRNs in a computationally efficient manner.
- By providing results from extensive simulations, we demonstrate that the proposed scheme achieves an improved beamforming gain as compared to a random node-selection scheme—the scheme in which nodes are selected at random— especially at low SNR conditions.

II. System Model and Assumptions

Consider an underlay CRN as shown in Figure 1, where SUs coexist harmoniously with other co-channel PUs without causing harmful interference to the latter. All PUs and SUs are assumed to be stationary, and each of them is equipped with a single omni-directional antenna. All nodes transmit with equal power. In our system model, only one SU-Tx cannot communicate directly with a SU-Rx either because of transmission-power constraint or because doing so will result in harmful interference to the co-channel PUs. All nodes agree to participate in cooperative beamforming, whenever needed. Although we consider only one PU in this work, our model can be easily extended to cases where there are multiple PUs.

Let us assume that there are N_T nodes in a CRN that are distributed randomly (following a uniform distribution) in a circle of radius R. As shown in Figure 2, a SU-Tx communicates with a SU-Rx by cooperating with other nodes to steer the signal towards the SU-Rx, while creating a null at the PU receiver (PU-Rx). Assume that only N nodes can collaborate at any given time due to system constraints such as synchronization and timely information-sharing. Therefore, the goal is to select the best set of N nodes among N_T available nodes for performing cooperative beamforming.

We assume that the nodes that participate in cooperative beamforming are controlled by a centralized node or through an ad-hoc network so that they can share their locations and other relevant information such as channel state information (CSI). Cooperative nodes obtain direction estimates of PUs and SU-Rx either through sensing, or by querying a geolocation database that provides the necessary information. For realizing beamforming, a channel that provides necessary angular/spatial separation between the directions of SU-Rx and PUs is chosen with the aid of a geolocation database. No particular assumption is made regarding the technology used by PUs and SUs.

A polar coordinate system is used to define the node positions where the origin is taken at the center of circle in which the nodes are distributed. The position of the n^{th} SU node is represented as (d_n, ϕ_n), where $n = 1, 2, \cdots, N_T$, d_n and ϕ_n are the distance and angle of the node from the center of the circle. The positions of PU-Rx and SU-Rx are represented as (d_{PU}, ϕ_{PU}) and (d_{SU}, ϕ_{SU}) respectively. The distance of the n^{th} node from the primary and secondary receivers are denoted by p_n and s_n respectively.

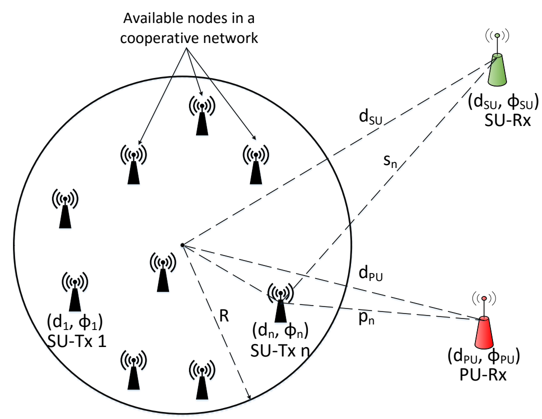

Figure 1. System model.

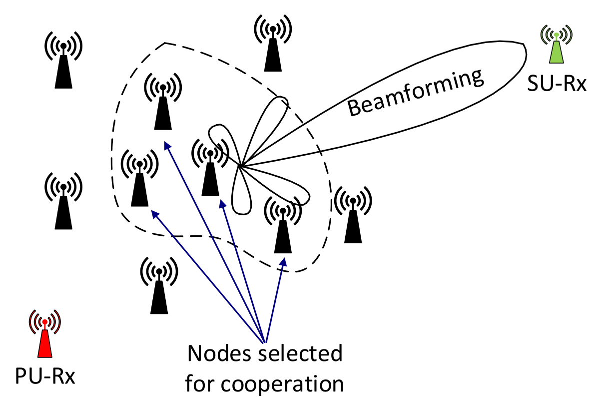

Figure 2. Desired beampattern with node selection.

The received SNR at the SU-Rx is used as a metric to evaluate the performance of the proposed scheme. Our objective is to maximize the SNR at SU-Rx while ensuring that the interference caused to the PU does not exceed a predefined threshold. For radio wave propagation, a simplified path loss model with log-normal shadowing is considered. Beyond a reference distance d_0, the path loss in dB (P_L) between two points separated by a distance d is given by,

$$P_L = a + blog_{10}d + \psi_L, \tag{1}$$

where $a = P_L(d_0) - blog_{10}d_0$, $P_L(d_0)$ is the path loss at the reference distance in dB, $b = 10\gamma$, where γ is the path loss exponent, and ψ_L is the log-normal shadowing coefficient with mean 0 and variance σ_L^2.

III. Cooperative Beamforming

A steering vector is defined as a representation of the set of delays that a plane wave, originated from the antenna elements of distributed SU-Txs, experiences in reaching a point in space. Based on the system model in Figure 1, for a given set of cooperating nodes, the steering vector towards a point (d, ϕ) is given as,

$$\boldsymbol{\alpha}(d, \phi) = [exp\left(-j\omega a_1/c\right), \ldots, exp\left(-j\omega a_N/c\right)] \quad (2)$$

where $\omega = 2\pi f_c$, f_c is the carrier frequency in Hz, and a_i is the distance of the i^{th} cooperating node, $i = 1, 2, \cdots, N_T$ from the point (d, ϕ).

The distance a_n can be computed as follows,

$$a_n = \sqrt{d^2 + {d_n}^2 - 2dd_n \cos\left(\phi - \phi_n\right)}. \quad (3)$$

Assuming $d \gg d_n$, a_n can be approximated as,

$$a_n \approx d - d_n \cos\left(\phi - \phi_n\right). \quad (4)$$

The channels from all cooperating nodes are assumed to be independent Rayleigh fading channels. Let the channel coefficient from an i^{th} cooperating node, $i = 1, 2, \cdots, N$, towards SU-Rx and PU-Rx be h_{s_i} and h_{p_i} respectively. Let $\boldsymbol{h_s} = [h_{s_1}, h_{s_2}, \cdots, h_{s_N}]$, and $\boldsymbol{h_p} = [h_{p_1}, h_{p_2}, \cdots, h_{p_N}]$. If the beamforming weight for an i^{th} node is w_i, and let $\boldsymbol{w} = [w_1, w_2, \cdots, w_N]$, then the received signal at SU-Rx is given by,

$$y_s = \boldsymbol{H_s w} x + \psi_s \quad (5)$$

where $\boldsymbol{H_s} = diag(\boldsymbol{h_s})\boldsymbol{\alpha_s}$, where diag($\boldsymbol{h_s}$) is a diagonal matrix of size $N \times N$ with elements of $\boldsymbol{h_s}$ as the diagonal elements, $\boldsymbol{\alpha_s} = \boldsymbol{\alpha}(d_{SU}, \phi_{SU})$ is the steering vector towards SU-Rx, and ψ_s represents additive white Gaussian noise with zero mean and variance σ_s^2.

Similarly, the interference signal received by PU-Rx, y_p, from cooperative transmitter cluster is,

$$y_p = \boldsymbol{H_p w} x + \psi_p \quad (6)$$

where $\boldsymbol{H_p} = diag(\boldsymbol{h_p})\boldsymbol{\alpha_p}$, where diag($\boldsymbol{h_p}$) is a diagonal matrix of size $N \times N$ with elements of $\boldsymbol{h_p}$ as the diagonal elements, $\boldsymbol{\alpha_p} = \boldsymbol{\alpha}(d_{PU}, \phi_{PU})$ is the steering vector towards PU-Rx, and ψ_p represents additive white Gaussian noise with zero mean and variance σ_p^2.

Let the interference tolerance threshold of the PU-Rx be I_P. Then, the cooperative beamforming problem can be formulated as the following optimization problem to find the optimum beamforming weights.

$$\begin{aligned} \text{Maximize} \quad & \left|\boldsymbol{H_s w}\right|, \\ \text{subject to} \quad & \left|\boldsymbol{H_p w}\right| \leq I_P, \\ & \left|\left|\boldsymbol{w_i}\right|\right|^2 = 1. \end{aligned} \quad (7)$$

The first constraint ensures the protection of PU from SU-induced interference. The second constraint on the weight vector implies that the total transmission power of each cooperating SU node is a constant.

Optimization problem (7) is a non-convex problem. In order to simplify the computations, the following equivalent problem can be obtained through some mathematical manipulations [5].

$$\begin{aligned} \text{Maximize} \quad & \Re\left\{\boldsymbol{H_s w}\right\}, \\ \text{subject to} \quad & \left|\boldsymbol{H_p w}\right| \leq I_P, \\ & \left|\left|\boldsymbol{w_i}\right|\right|^2 = 1, \\ & \Im\left\{\boldsymbol{H_s w}\right\} = 0, \end{aligned} \quad (8)$$

where $\Re\{X\}$ and $\Im\{X\}$ are the real and imaginary parts of X respectively. For a given set of cooperating secondary nodes, the optimization problem (8) can be solved using any *second-order cone programming* (SOCP) solver.

IV. Node-Selection Schemes

In the previous section, we formulated, for a given set of cooperative nodes, an optimization problem to compute the optimal beamforming weights. Since different nodes have different channel states towards the intended SU-Rx and the PU, the selection of nodes affects the performance of cooperative beamforming, and hence, an arbitrarily chosen set of nodes may not be able to achieve the best beamforming performance. In this section, we discuss several node-selection schemes in a cooperative network, and introduce our proposed scheme that achieves a near-optimal performance with very low computational complexity.

A. Random Selection

In our system model, the total number of nodes in the network is N_T and only N among these nodes can participate in cooperative beamforming. Therefore, there are a total of $\binom{N_T}{N} = \frac{N_T!}{N!(N_T-N)!}$ node combinations. The simplest approach to select the nodes would be to randomly choose one among $\binom{N_T}{N}$ combinations, and solve the optimization problem (8) for that set of nodes and corresponding parameters. However, as expected, the performance of this scheme is poor, especially at low SNR conditions.

B. Nodes with Highest Channel State towards SU-Rx

An intuitively better approach to select N nodes from N_T nodes would be to select the N nodes that have the maximum value of channel state in the direction of SU-Rx, i.e., select the nodes with maximum value of $|H_{s_i}| = |h_{s_i}\alpha_{s_i}|$, where α_{s_i} is the i^{th} element of $\boldsymbol{\alpha_s}$. This set of nodes is an ideal candidate for cooperative beamforming in the absence of any PU. However, in presence of PUs, satisfying the first constraint of (8) may produce beamforming weights that result in small gain towards SU-Rx even when the channel state towards SU-Rx is very good.

C. Exhaustive Search

Our detailed analysis suggests that only an exhaustive search through all $\binom{N_T}{N}$ node combinations is able to find the optimal set of nodes that achieves the best beamforming performance in a cooperative CRN. The idea of an exhaustive search is to solve the optimization problem (8) for each node combination, and select the set of nodes that offer the best beamforming gain. While this may be feasible for small system sizes, the number of computations increase quickly when N_T and/or N increase. For example, with $N_T = 10$ and $N = 5$, the optimization problem needs to be solved 252 times. On the other hand, with $N_T = 20$ and $N = 5$, the optimization problem needs to be solved $15,504$ times. It must be noted that a set of nodes may be optimal only for a particular set of channel state at the cooperating SU-Txs. If the channel at any SU-Tx changes, the nodes may not be optimal any longer.

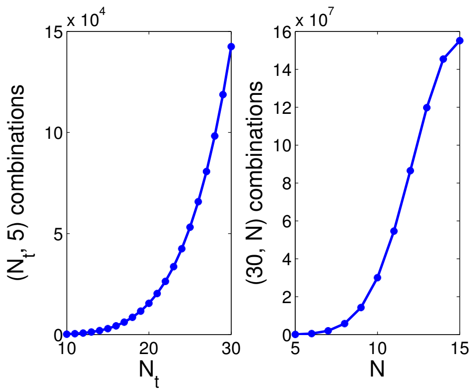

Figure 3. Computations required for exhaustive search based node selection.

Figure 3 shows the number of combinations versus N_T and N. It is clear that the computation complexity quickly becomes prohibitively high as N_T and/or N increases. Thus, performing exhaustive search in real-time is not feasible, and there is a need for a node-selection scheme that offers a near-optimal beamforming performance, and is computationally feasible to implement in real-time wireless networks.

D. Proposed Scheme

As mentioned before, the gain achieved by cooperative beamforming at the SU-Rx primarily depends on the choice of participating nodes. Our detailed analysis reveals that a particular node may perform poorly with an arbitrary set of $N-1$ nodes, but the same node may perform extremely well with some other set of $N-1$ nodes that are selected from the remaining $N_T - N$ nodes. Thus, even if a given node performs poorly with an arbitrary set of nodes, it cannot be concluded that the node is not one of the optimal nodes. This insight forms the basis of our proposed algorithm. The proposed node-selection scheme selects a near-optimal set of nodes—i.e., the set of nodes that performs very close to the optimal set of nodes. The big advantage of the proposed scheme is the reduction in computational complexity which makes it scalable to large CRNs, where effective node-selection is crucial to the performance of cooperative beamforming.

Let us consider, for example, there are 12 nodes in the network, i.e., $N_T = 12$, and system constraints allow only $N = 5$ to participate in cooperative beamforming. Then, the total possible combinations of cooperative nodes are $\binom{12}{5} = 792$. Suppose 100 combinations, out of these 792 combinations, perform relatively good (say within 5 dB of the optimal performance), while others perform bad (lower than at least 5 dB of the optimal performance). Now, if a set of 5 nodes are picked at random, the probability that the performance of cooperative beamforming is bad is $\frac{692}{792} = 0.8737$, which is very high. However, because of birthday paradox, if we pick a set of 5 nodes independently 20 times and select the set of nodes that performs the best among the 20 combinations, then the probability that the performance of cooperative beamforming is bad is $\left(\frac{692}{792}\right)^{20} = 0.0672$, which is a very less probable event.

We consolidate the above two insights and propose a computationally efficient node-selection scheme for cooperative beamforming. First, a set of randomly chosen N nodes, along with their channel state towards SU-Rx and PU, is used to solve the optimization problem 8. For this set of nodes, the optimal beamforming weights, as well the corresponding beamforming gain, is computed. Next, the node that contributes the minimum towards cooperative beamforming among the N nodes is identified. The minimum contributing node is the one that has smallest value of $|H_{s_i} w_i| = |h_{s_i} \alpha_{s_i} w_i|$. Then, the algorithm replaces the lowest contributing node with an another node that is picked randomly from the remaining $N_T - N$ nodes.

The aforementioned steps are repeated I_{th} number of times, and the algorithm picks the set of nodes that performs the best among all I_{th} sets. Note that a node that was discarded in the earlier iteration may be repicked in the next iteration as it could perform well with the new set. Here, I_{th} is a threshold parameter that determines the complexity of the algorithm, as well as its performance. Small I_{th} corresponds to less computations, but it may result in poor beamforming performance. Thus, there is a trade-off between computational cost and beamforming gain. The system can be designed to achieve a balanced trade-off between these two parameters. Our results, which are presented in the next section, show that a near-optimal beamforming performance can be achieved within few iterations—i.e., with a small I_{th} value. Moreover, the computational complexity of the proposed algorithm is agnostic of the network size, N_T and N, which makes this approach attractive for large networks. The proposed algorithm is summarized in Algorithm 1.

V. Simulation Results

In this section, we provide detailed simulation results to demonstrate the performance of the proposed node-selection scheme for cooperative beamforming. Assume that $N_T = 15$ cooperative nodes—each equipped with a single omni-directional antenna—are distributed uniformly in a circular area of radius, $R = 75$ meters. Only $N = 5$ among N_T nodes can collaborate for performing cooperative beamforming. The intended SU-Rx is located in a direction of $30°$ from the reference direction and at a distance, $d_{SU} = 1$ km. The PU-Rx is located in a direction of $80°$ from the reference direction at a distance, $d_{PU} = 1$ km.

Algorithm 1 Node-Selection Algorithm

1: Select threshold variable I_{th}.
2: Choose a random set of nodes N from N_T nodes, and assign it as the 1^{st} set of nodes.
3: **for** i in $\{1 \cdots I_{th}\}$ **do**
4: Solve optimization problem (8) for i^{th} set of nodes.
5: Sort the nodes in ascending order of index, and save them, along with the corresponding objective-function value, as an i^{th} vector of a vector-list $\boldsymbol{M}$.
6: Find a node, say n_i, from the i^{th} set that contributes the minimum towards the objective function (i.e. $|\boldsymbol{H_s w}|$).
7: Replace n_i with a node chosen randomly from the remaining $N_T - N$ nodes.
8: **if** the new node-combination has already been selected before (i.e., if it is present in the vector-list $\boldsymbol{M}$), **then**
9: Go back to step 7.
10: **end if**
11: **end for**
12: Return the set of nodes that perform the best among all I_{th} sets.

The following RF parameters are used throughout our simulations: $f_c = 900$ MHz, $\gamma = 2.5$, $\sigma_L = 3$ dB and $I_P = -80$ dBm. The transmission power of each SU node is $P_{tx} = 23$ dBm, and the noise level is $\sigma_s^2 = \sigma_p^2 = -105$ dBm. Each node experiences an uncorrelated Rayleigh fading channel in all 360° directions. For simulations, the channel is generated independently in each direction.

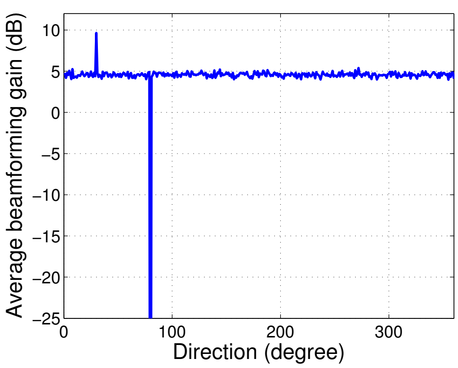

Figure 4. Performance of cooperative beamforming

A. Performance of Cooperative Beamforming

The performance of cooperative beamforming is evaluated by computing the average beamforming gain, $|\boldsymbol{H_s w}|$, over 500 simulation runs, where $\boldsymbol{w}$ is obtained by solving optimization problem 8. The locations of N cooperative nodes are generated randomly in different simulation runs. Figure 4 shows the result. Clearly, the beamforming gain towards the SU-Rx is maximized, and a null is generated in the direction of the PU-Rx. The sharp peak and the null towards SU-Rx and PU-Rx respectively are consequences of the uncorrelated channel assumption (recall that, for each node, we assumed independent channel in different directions), and it verifies that the cooperative beamforming considered in this study achieves the dual objectives of a cognitive radio network—(i) maximize the signal at the intended receiver, and (ii) minimize interference at the PU-Rx.

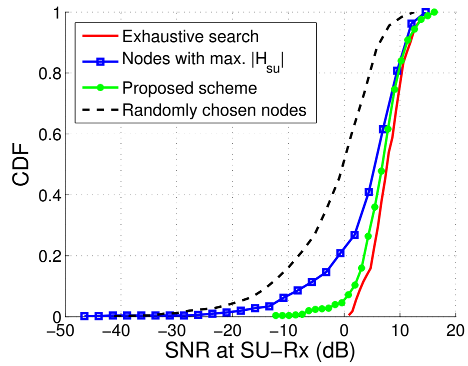

Figure 5. CDF of SNR at the SU-Rx ($N_T = 15, N = 5$)

B. Analyzing the Performance of the Proposed Scheme

Here, we analyze the performance of the proposed scheme and compare it against all other schemes discussed in Section IV. Specifically, we compare the cumulative distribution function (CDF) of SNR at SU-Rx and the CDF of received signal strength (RSS) at PU-Rx for different node-selection schemes. Specifically, we set $N_T = 15$, and $I_{th} = 10$. The results are summarized in Figures 5 and 6.

Figure 5 shows the CDF of SNR at SU-Rx. We can notice the performance of random node-selection scheme is the worst while the performance of the exhaustive search is the best among all schemes. The scheme that chooses nodes with maximum $|H_{su}|$ performs better than random node-selection scheme, but it is not as good as the proposed scheme. Interestingly, the performance of the proposed scheme is comparable to that of exhaustive search. Note that, for this simulation set up, exhaustive search requires $3,003$ iterations while the proposed scheme achieves the reported performance in only 10 iterations ($I_{th} = 10$). Figure 6 shows the CDF of interference power at the PU-Rx. We observe that the proposed scheme, as well as others, satisfies the protection requirement of the PU-Rx—i.e., the proposed scheme is able to constrain the interference power at PU-Rx below the predefined interference tolerance threshold, $I_p = -80$ dBm. In summary, our results validate the efficacy of the proposed node-selection scheme in achieving a near-optimal solution for node-selection in cooperative beamforming. The significantly low computational complexity is the most attractive feature of the proposed scheme.

C. Asymptotic Performance of the Proposed Scheme

In this section, we use simulation results to analyze the asymptotic performance of the proposed node-selection scheme. For performing this study, we set N_T to 15. Figure 7 shows the CDF of SNR at SU-Rx. We observe that, as we

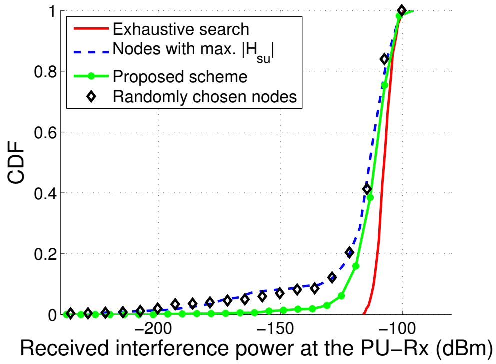

Figure 6. CDF of interference power at the PU-Rx ($N_T = 15, N = 5$)

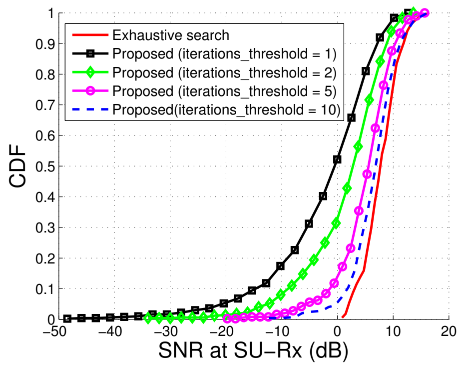

Figure 7. Performance comparison: proposed scheme vs. exhaustive search

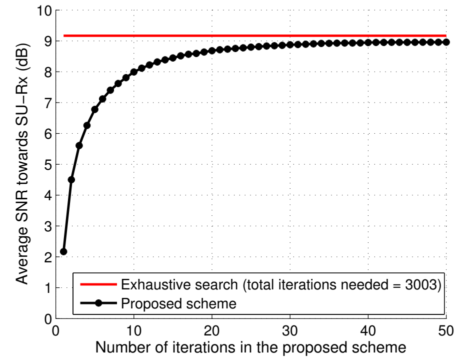

Figure 8. Asymptotic performance of the proposed node-selection scheme

increase the number of iterations (I_{th}) in the proposed scheme, the performance of the proposed scheme closely resembles that of exhaustive search. Intuitively, for $I_{th} = 1$, the proposed scheme is equivalent to the random node-selection scheme. However, as we increase I_{th}, the CDF curve shifts rapidly to the right (towards the optimum case). This provides us an insight about the relation between I_{th} and the performance of the proposed scheme—with an increase in I_{th}, the proposed scheme can perform as good as the exhaustive search. Figure 8 validates this claim. Clearly, the asymptotic performance of the proposed scheme quickly approaches the limit (the performance of the exhaustive search), and it follows the law of diminishing returns. In our example, the performance of the proposed scheme is approximately 98.5% of the optimal system performance with only 1.67% computations.

VI. Conclusions

In this paper, a computationally efficient node-selection scheme is proposed for cooperative transmit beamforming in a cognitive radio network. The proposed scheme provides a near-optimal beamforming solution while significantly reducing the computational burden. Specifically, a reduction in complexity from $\binom{N_T}{N}$ to I_{th} is achieved, where $I_{th} << \binom{N_T}{N}$. Our simulation results show that the performance achieved by the proposed scheme is approximately 98.5% of the optimal solution with only 1.67% computations when $N_T = 15$ and $N = 5$. Our proposed solution will be effective in choosing participatory nodes for cooperative beamforming in large wireless networks.

VII. Acknowledgment

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